MONTE CARLO SIMULATIONS OF PRODUCTION COSTS IN DISCRETE MANUFACTURING

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Abstract: When calculating the production costs input data can vary considerably in quality between different cases. When costs pertain from products, which are already in production, complete and reliable data may already be available. For new products, new production systems or new production technology it might be hard to find applicable and reliable data. The cost and performance parameters needed can then be achieved by examining similar products or processes, these parameters will inevitably differ from the new case. The aim of this paper is to try to Monte Carlo simulate production costs, with less reliable data.

Keywords: Monte Carlo Simulation, Production cost, New Technology

1. INTRODUCTION

In today's competition it is of outer most importance to be able to calculate the production cost. The analysis of production costs can be used for steering the development of the production and products, it can also be used in prizing of the product. The production cost depends on many different parameters, such as; material cost, machine costs, wage costs, down times, quality losses etc. It can sometimes be hard to find reliable data in the industry to thoroughly calculate the costs of production. With the exception of the automotive industry it is not uncommon with industry missing important data due to lack of process monitoring. Down times and quality problems have often been seen, by the authors, to be recorded manually. When collecting data manually smaller disturbances might be overlooked, due to the fact that the operators deem them negligible. As shown by Stål *et al.* (2012) these disturbances can still have a big influence on the part cost when accumulated. Parameters such as market demand, down time rate and quality losses also have a variation. If the cost then is calculated taking into account the variation of the different parameters the cost will vary.

The Monte Carlo Method was first presented by Metropolis and Ulam (1949) to simulate particles in a nuclear reactor. In their initial paper on the Monte Carlo Method, Metropolis and Ulam (1949) write about using several computers to simulate data. Today the computer power, as is well known, is much greater than the one at that time. To simulate a random vector with a 1000 values in a computer program does not even take one second on a standard laptop. In the last decades the Monte Carlo method has been used in several different fields, within the mechanical engineering field Chih-Young *et al.* (1995) used it to simulate tolerances for assembly. In the energy sector Monte Carlo simulations is commonly used to model power production costs some examples are given by Chiang, J-Y *et al.* (2000).

Ståhl *et al.* (2007) developed a cost model from here on referred to as Ståhls model, this study is built on that model. Jönsson (2012) has done an extensive literature study of different cost models, the results of this study shows that there are not many generic models with the level of detail desirable for this study. Some requirements set was that the model used should include as many different cost and performance parameters as possible but be limited to the actual manufacturing part of the factory. In this paper the goal is to try and simulate the different cost parameters in Ståhls model. This will be done by Monte Carlo simulations in its simplest form. The aim is to be able to simulate the production cost with limited data collection. With this limited data a statistical function will be extracted through Monte Carlo simulations. Untzeitig *et al.* (2014) has used a similar approach to reduce the uncertainty if the early phases of production planning. Some of the parameters that Untzeitg uses are the same as in Ståhls model.

2. GOALS, METHOD, SIMULATIONS

The established Ståhls model, see equation 1, is used to study the parameters; required production time, capacity and production cost and show their statistical distribution in an illustrative way. The model used should be generic so that it can be used for many different types of production. It ought to show which parameters that have the most influence on the cost to see which parameters to focus on. The developed model shall be able to be used as a decision support for:

- Strategic Technology decisions with regards to cost and competitiveness for:
  - A new Production process for an existing product
  - A new Product in an existing Production process
  - A new Product in a new Production process, at least if the new product and/or process is similar to a, known, existing product/process
- To see the potential in an existing process with an existing product through scenario description of the development of existing process
- Support on pricing of the product

With existing product the author means a product that is already in production.

The theory is that Monte Carlo simulations can be used to simulate the different cost and performance parameters in Ståhls model (Ståhl *et al.* 2007), see equation 1 and APPENDIX for list of parameters. The Monte Carlo method is calculations done on random numbers, it's an experimental method (Hammersley, 1960). The method is based on observed data. The parameters in Sthåls model should after simulation be able to be used to calculate a possible cost range to produce the product. The outcome of a Monte Carlo simulation is in this case a randomised vector for each simulated parameter, the values in the vectors will have predetermined statistical distributions. To create these vectors a domain of possible inputs needs to be defined and a probability distribution needs to be decided upon for each parameter. The computations are then done in a mathematical computer program, in this study Mathcad. To decide the domain of possible values, data from existing processes, machine supplier and estimations from experienced personal and engineers can be used. The end result of the simulations should be a distribution that shows the best case, the worst case and a likely cost to produce one part.

$$k = \frac{k_A}{N_0}\left[\frac{N_0}{(1-q_Q)}\right] + \frac{k_B}{N_0}\left[\frac{N_0}{(1-q_Q)}\right] + \frac{k_{CP}}{N_0 60}\left[\frac{N_0 t_0}{(1-q_Q)(1-q_P)}\right] + \frac{k_{CS}}{N_0 60}\left[\frac{N_0 t_0}{(1-q_Q)(1-q_P)} \cdot \frac{q_S}{(1-q_S)} + T_{su}\right] + \frac{k_D}{N_0 60}\left[\frac{N_0 t_0}{(1-q_Q)(1-q_P)(1-q_S)} + T_{su}\right] \quad (1)$$

A small case study was made at a Swedish company to illustrate the method. There is no focus on the result of the case study other than for testing the method. The company was asked to provide maximum and minimum values for the different cost and performance parameters and the probability of exceeding these values, see example in table 1. In this case it was the production engineer at the company, together with experienced operators that did the estimations of the values. For a more in depth study of existing processes more people should be interviewed. For some data, for example interest rates, other employees than engineers or operator should be interviewed, it can be employees from the economics department, human resources, etc.

Table 1, Example of data collected.

Parameter	Maximum value	Probability	Minimum value	Probability
Setup time	3h	95%	45min	99%
Down time rate	25%	90%	10%	95%

With this data an extensive computer model is setup to model every parameter in Ståhls model. Some of the parameters in Ståhls model persist of several parameters. For example: k_{CP}, [machining cost/hour] which depends on cost for buying the machine, interest rates, number of years the machine is used, the area the machine occupies, process additives etc. see Jönsson (2012, page 61). The exact form of the model is shown later on in this paper.

In a Monte-Carlo simulation, the same number of data points needs to be available for estimating the value of each variable. At the same time, for practical reasons, a statistical assessment of the value of different variables can be based on bodies of data of differing size. One method of obtaining bodies of data of equal size for each variable is to determine a constant for each of the distribution functions involved and then to randomly generate a vector of the length needed for each of them.

Figure 1 is a visualization of how many simulated data points per vector is needed to get a ”good” result. The variable n is the number of simulated data points. As can be seen the curve smooth’s out around n=1000 data points. To be on the safe side n has been set to 2000 data points in the simulation tests. With today’s computer power the extra time to calculate the total cost with the number parameters used is negligible.

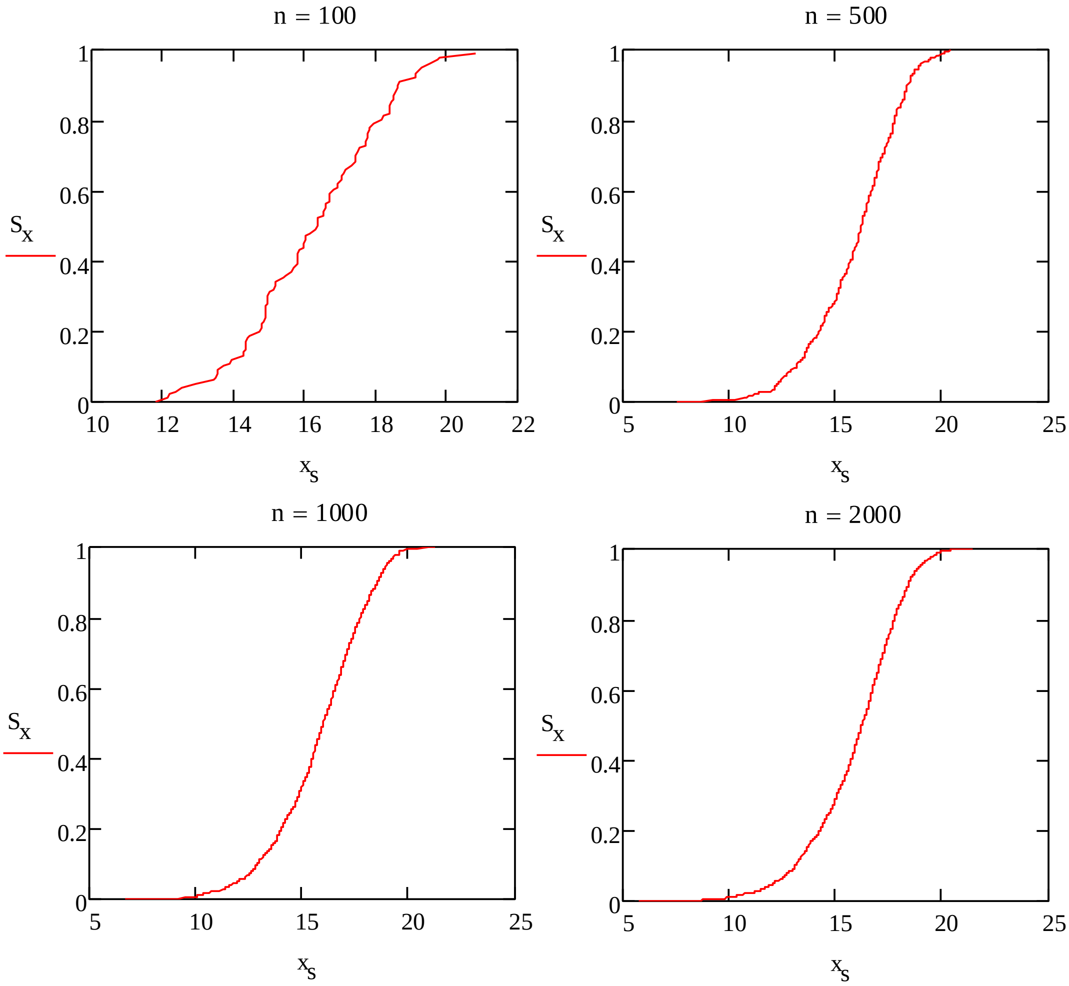

Fig. 1. Example of cumulative distribution functions for different number – n of input values.

In this first modelling of the method each parameter in equation 1 is assumed to have a simple distribution. The method should be able to be used for mixed distributions as well. In the case study the parameters are also assumed to be independent of each other even though as Windmark *et al* (2014) suggests that is not always the case. To simplify the mathematics only weibull distributions has been used in this first test.

3. SIMPLIFIED MONTE CARLO

Monte Carlo simulations can be fairly complicated. In this study it has been simplified and the taken approach can be seen schematically described in figure 2. The approach is as follows; first the parameters maximum and minimum values are set, the domain of values will be between these numbers and have a certain probability that is also set. Through simulation using the least square method the values of the α and β parameters in equation 6 are then generated. A random vector with values between zero and one is created and the distribution function is set. With the use of the α and β parameters and the randomized vector a new vector with the set distribution function is set.

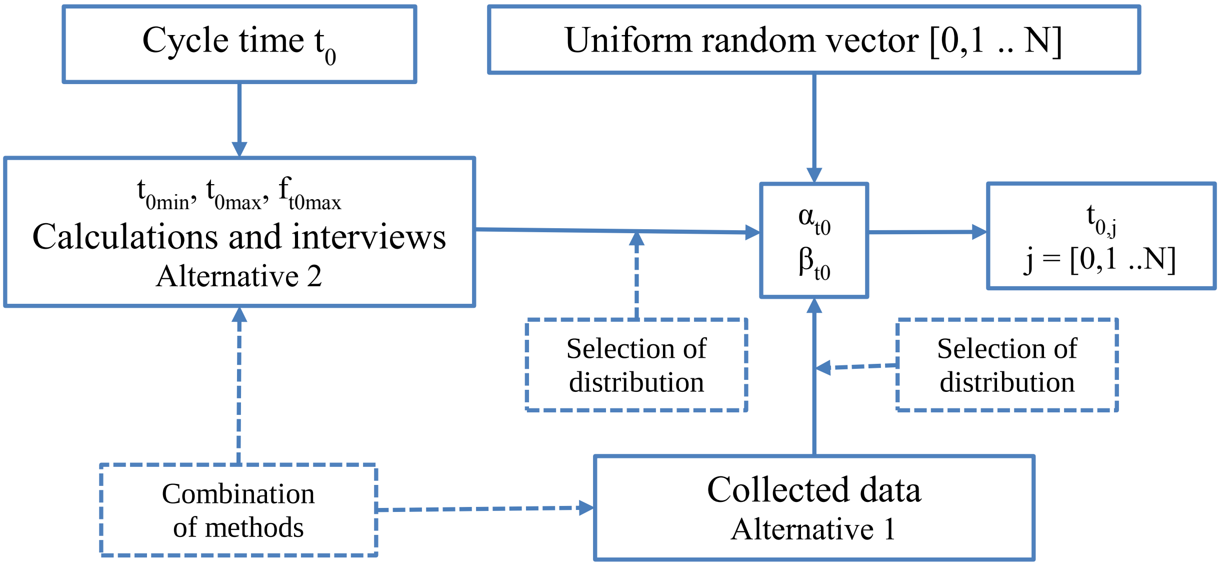

Fig. 2. An example of how a randomly generated data vector t_0 of length N+1 can be created for each of a set of known constants α_{t0} and β_{t0}.

To do the simulations the equation system equation 2-6 has been used. The parameters p_{max} and p_{min}, in equation 2 and 3 are given the estimated maximum and minimum values of the parameters and G_{max} and G_{min} the probability of those values as described earlier. By the use of the least square method and the equations 2 and 3 the values of the statistical parameters α and β are generated. Equation four is a build in function, in Matchad, to generate random numbers. In this case it will create a vector with the length n_R with random numbers between zero and one. Equation five creates a range variable with numbers from zero to n_R-1. Equation six adapts the generated vector p_W to a weibull distribution with the simulated values of α and β.

$$p_{max} = \beta(-ln(1 - G_{max}))^{\frac{1}{\alpha}} \quad (2)$$

$$p_{min} = \beta(-ln(1 - G_{min}))^{\frac{1}{\alpha}} \quad (3)$$

$$p_w := runif(n_R, 0,1) \quad (4)$$

$$j := 0..last(p_W) \quad (5)$$

$$p_j = \beta_p \left(-ln(1 - p_{W_j})\right)^{\frac{1}{\alpha_p}} \quad (6)$$

Figure 3 illustrates an example of a distribution function for the generated vector p_j. The input values in the example is p_{min}=0.07, p_{max}=0.12, G_{min}=0.05 and G_{max}=0.95. To create the graph the values are sorted in order and then plotted.

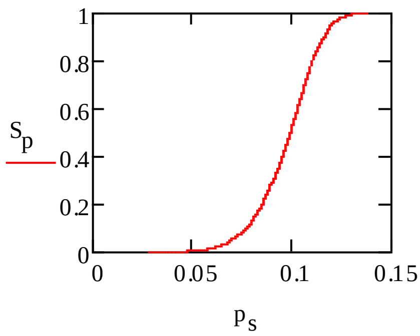

Fig. 3. Example of distribution function for generated values.

Equation 7 illustrates an example for a part of the cost model. The machine cost, during production, per part is calculated. The arrows are used to represent the vectorization of the different input parameters. The parameters under the arrows, with exception of k_{CP}, are vectors that are generated through simulation as in equation 2-6. The parameter k_{CP} is simulated in depending on several variables as earlier mentioned. The outcome of equation 7 is also a vector representing the machining cost per part.

$$k_{CP}/pcs = \frac{\overrightarrow{k_{CP}}}{\overrightarrow{60N_0}}\left[\frac{\overrightarrow{N_0 t_0}}{\overrightarrow{(1-q_Q)}}\right] \qquad (7)$$

Figure 4 is the probability density function of the machining cost per part in the case study. As can be seen it reaches from around 2 to 3.8 units of monetary value. The black line in the figure represents the mean value and is just below 3 units in monetary value. This representation could be done for all the parameters simulated.

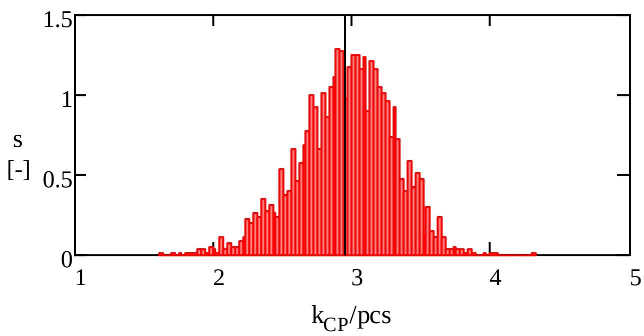

Fig. 4. Probability density function for machining cost per part.

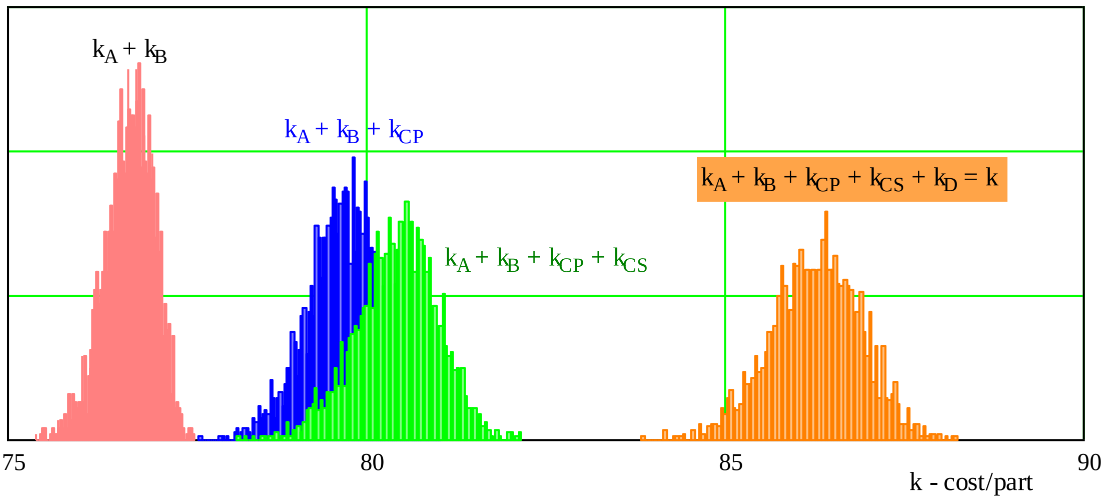

Fig. 5. The probability density functions for accumulated values of the different parts of Ståhls model.

The simulated parameters in equation 1 are then calculated as the example in equation 7 and figure 5. The cost per part for k_A/part - tool cost, k_B/part – raw material cost etc. is separated to be able to see how much each parameter varies and how much it costs in monetary value. In figure 5, the costs are then added one by one to make the total cost. The probability density function to the right in the figure is the total cost for producing one part in the studied production process. As can be seen the cost varies from approximately 85 to 87.5 in monetary value.

4. CONCLUSIONS AND DISCUSSION

In an existing production process with an existing product both the initial investment of the machines and the material cost are known fixed values. With a new production process and/or a new product the initial investment in machines and the material cost might not be known. The estimation of the different process parameters such as down times and wear rate of tools are more difficult with a less known process. The probability to stay within the estimated minimum and maximum values will be lower for new processes then for old established processes. This means that the result of the total cost will have a bigger variation for a new process then for an existing one. At the different steps of ramp up of a new product or production process the values in the simulation can be updated with more information to make the cost more reliable and variation in the simulated cost smaller. To get an understanding of which parameters have the biggest influence on the cost the different parameters could be simulated with the other parameters set to fixed or nearly fixed values.

The studied case is a well-known process that has been in use for more than five years. That is probably why the variation in cost is quite small, less than 3%. The variation of each input variable is small and the probability to fall outside the estimated maximum and minimum values is considered small since the process is well known. Also the material cost, which is the largest cost at over 70% of the total cost, is a fixed cost. In cases such as this one where the process is well known the simulations can be used to see the potential in the system. It is not likely to produce the product at a lower cost than that at the lower limit without doing investments in the process. When setting the prize for a known process with this small variation the worst case, with a certain probability, could be used.

When simulating the cost of a new product or process the variation in cost will be fairly high, initial tests from the same company as the one mentioned in the case study, has shown up to 200%. If this is the case more information needs to be collected before any decision can be made. When the investment cost is set with a good probability and small variation between the maximum and minimum values the variation can also be seen as the potential. In the beginning of production of a new product the risk of being closer to the higher cost simulated because the different performance parameters is likely to be higher. For example when starting to produce a new product the down time rates and quality losses are generally higher than in normal production. The cost is therefor likely to be closer to the worse case. As the baby diseases of the production disappear the cost should go towards the mean of the process and the potential could be seen at the lower end of the cost range. When setting the prize of a new product the simulation should be able to help by providing an initial idea of what the cost could be.

These types of simulations could be applied on the other cost models as well. On cost models with parameters and variables that vary with certain distributions. Other mathematical programs or programing directly in some computer languages can also be used to do the simulations.

The major problem with these kinds of simulations is that the distribution function needs to be known for each parameter. This means that some measurements on the process at hand needs to be done. In cases of new products and new processes experience from similar processes could be used to set the distribution functions. To confirm the model further tests needs to be done out in the industry. These tests should be designed so that the different parameters first are estimated by the industry personnel then simulated and then confirmed by actually measuring the parameters and calculating the actual cost. Tests with dependent parameters should also be done.

5. ACKNOWLEDGEMENT

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APPENDIX A: LIST OF PARAMERTERS

Parameter	**Description**	**Value**
k_A	Tool cost per part	currency/unit
k_B	Material costs per part	currency/unit
k_{CP}	Hourly machine costs during production	currency/h
k_{CS}	Hourly machine costs during downtimes and adjustments	currency/h
k_D	Salary costs	currency/h
N_0	Nominal batch size	unit
q_P	Production-rate loss	-
q_Q	Rejection rate	-
q_S	Downtime rate	-
t_0	Nominal cycle time per part (for line production the through-put time)	min
T_{su}	Set up time	min