The tool life model formulated by Bertil Colding is limited to the use of a specified tool life criterion. The Colding equation describes the relationship between tool life, cutting speed and the equivalent chip thickness. The Colding equation is based on five constants which have unique values for each selected value of the tool life criterion. This paper investigates how these Colding constants can vary for different selected wear criteria.

Keywords: Machining, Turning, Tool life, Wear model, Equivalent chip thickness.

1. INTRODUCTION

It is of significant importance to be able to predict and describe the tool life and as a result determine optimal cutting data in regards to manufacturing cost during industrial production. When estimating the optimal cutting data for example in terms of minimizing the part cost, analytical models for describing the tool life in relation to the cutting data is needed. Cutting data commonly involve the cutting speed \( v_c \), feed \( f \) and depth of cut \( a_p \). These three parameters when multiplied will quantify the removal rate, commonly called MRR (Metal Removal Rate). The engagement time \( t_i \) divided by the MRR is directly proportional to the machining cost. The tool life \( T \) describes the length of time the tool can be engage with the workpiece without being worn out in respect to such parameters as e.g. risk of tool failure, substandard surface quality or product dimensions outside the given tolerances. A wear criterion is commonly decided with regards to these factors, such as a maximum allowed size of the flank wear, i.e. \( VB < 0.3 \text{mm} \).

A wear model describes the relation between the engagement time \( t \) and the attained tool wear such as flank wear \( VB \) for different cutting data or cutting conditions. Examples of previously published tool wear models include models published by Archard (1953) and Usui, et. al. (1984). Examples of tool life models have been published by Taylor (1906) and Colding (1959). Tool wear models like those published by Archard (1953) and Usui, et. al. (1984) contains hard to determine constants that are determined as results from mechanical, thermal, tribological and chemical loads. The models may well work in controlled and well-defined laboratory conditions. The difficulty of these models is the adaptability and validity under industrial conditions. Models describing the tool life as a function of cutting data according to Taylor (1906) and Colding (1959) require that test are made until the tool wear criterion is fully reached. The experiments will then have to be repeated several times to reach statistical certainty. The minimum number of trials that must be performed is determined by the number of constants included in the given model. These models require access to high volumes of work material and the experiments are rather time consuming. The Colding equation has a high validity in many applications according to Hägglund (2013). This study shows, among other things that under favourable conditions the cutting speed \( v_c \) can be determined by the model with less than 1% error for a given tool life.
2. GOALS AND LIMITATIONS

The aim of the work presented in this article is to study the effect of different wear criterion on the Colding equation’s constants and how this affects the model error. Further, the influence on the constants in Colding’s tool life equation and the proportion of tool coating left on the tool for different wear criteria was studied and is presented as a part of the article. As the tool wear VB increases the coating will gradually be consumed and, as the coating ratio decreases in respect to the contact area, the properties of the substrate material will have an increasing importance during the machining process. Also, the coating ratio in the contact area and its effect on the model error was studied and is presented as part of the current article.

This research is limited to include longitudinal turning of a common carbon steel AISI 4340 (SS 2541). For this material, flank wear dominates, which is a prerequisite for the validity of the Colding model.

3. THE COLDING MODEL

The Colding equation is essentially based on empirical curve adjustments made between tool life and cutting data. The equations can be regarded as a development of Taylor’s more well-known equation (Taylor, 1906), which can be clearly observed in studies of Lindström’s reformulation of the Colding equation (Lindström, 1989).

The generalized Taylor equation, used by many, contains 3 empirical constants and produces straight, parallel curves in both the cutting speed-time (v_c-T) and cutting speed-chip thickness (v_c-h_e) planes, using log scales. Colding (1960) noted that when using this approach for a wide range of machining data, the accuracy of the tool life estimates were poor, except for the cutting data range it was based on. An ideal polynomial relationship was proposed by Colding in 1961 containing 9 constants. Colding later in 1981 presented a new equation containing 5 constants due to the enormous amount of tool-life tests needed for the latter, prohibiting its use. Five constants can be handled with a reasonable amount of tests. The three tool-life relationships with 3, 5 and 9 constants are presented below, Eq. 1-3 where x is theoretical chip thickness, y is cutting speed and z is time, all in log-log scale. The constants being a, b, c, d, e, f and k.

1. 9 Constants (Colding, 1961)

\[ k + y + ay^2 + bx + cx^2 + dz + ex^2 + fxy + gyz + hxz = 0 \]  

(1)

2. 5 Constants (Colding, 1981)

\[ k + y + bx + cx^2 + dz + hxz = 0 \]

(2)

3. 3 Constants (Taylor 1906)

\[ k + y + bx + dz = 0 \quad \text{where} \quad x = \ln h_e, \quad y = \ln v_c \quad \text{and} \quad z = \ln T \]

(3)

Five constants can be handled with a reasonable amount of work and this article, when referring to the Colding equation, thus refers to Eq. 2.

Colding’s equation for 5 constants can be rewritten in terms of a parabolic equation as presented in Eq. 4 which, for a given combination of cutting tool and workpiece, describes the relationship between the tool life T of the cutting tool, the cutting speed v_c and the equivalent chip thickness h_e.

\[ v_c = e^{K - \frac{(\ln(h_e)-H)^2}{4M} - \left(\frac{N_0 - L \ln(h_e)}{\ln(T)}\right)} \]

(4)

\[ T = e^{\frac{H^2 - 2H \ln(h_e) + \ln(h_e)^2 - 4K M + 4M \ln(v_c)}{4M \left(N_0 - L \ln(h_e)\right)}} \]

(5)

The Colding equation, Eq. 4, is according to Colding (1991) based solely on the curve adjustment of measurement points, without its having any clear relationship to the tool deterioration.

Woxén (1932) introduced an equivalent chip thickness h_e for turning operations with the purpose of using it as a characteristic parameter for describing the mean theoretical chip thickness along the tool nose.
\[ h_e = \frac{A}{l_e} \approx \frac{a_p \cdot f}{a_p - r(1 - \cos \kappa) + \kappa \cdot r + \frac{f}{2 \sin \kappa}} \]  

(6)

Additional relationships for calculating the equivalent chip thickness has also been published by among others Bus, et al. (1971), Hodgson, et al. (1981) and Carlsson et al. (2001). Equivalent chip thickness according Woxén is an approximation that provides issues when calculation for low values of feed. Ståhl, et al. (2012) have published an more accurate model which also is valid for finishing operations where feed and depth of cut can be of the same magnitude.

The Colding equation should be used with considerable caution outside the cutting data interval in which the measurement points are located. Sizeable errors can be made in estimates of tool life based on extrapolations from the cutting data available. Difficulties arise caused by numerical problems in calculations of Colding’s equation as the denominator of Eq. 5 assume a low value, close to zero. This singularity is obtained for a given value of the equivalent chip thickness \( h_e \) according to Eq. 7. Colding (1981) brings up this issue in the article “The machining productivity mountain and its wall of optimum productivity” stating that the model may not be reliable at long tool lives e.g. in excess of 300 minutes, as small removal rates at small feeds and cutting speeds may cause “build-up” and abrasion rather than cutting. This was observed further by Ståhl (1987) when introducing what he named as synthetic flank wear \( VB_s \).

\[ h_{es} = e^{\frac{N_0}{L}} \]  

(7)

Fig. 1. Expected wear propagation on a tool with synthetic flank wear \( VB_s \) (A) compared to a conventional tool (B), (Ståhl, 1987).

As Colding’s equation contains 5 constants \( C_i = (K, H, M, N_0, L) \), at least 5 separate experiments are needed to determine their values. Experience shows that each investigation should involve measurements being made on at least 2 cutting tools that yield the same equivalent chip thickness \( h_e \) and for which the same cutting speeds are employed. It can be sensible to select the measurement points in such a way that they are representative of the cutting data expected to be used for the cutting operation in question. Colding’s constants are preferably attained with the help of curve adjustment and use of methods aimed at keeping measurement errors at a minimum. The uses of Colding’s equation and adaption to different machining operations have been further investigated by Hägglund (2013).

4. EXPERIMENTS AND CALCULATION OF CONSTANTS

The experimental trials were conducted through longitudinal turning bars of AISI 4340 while using coated cemented carbide cutting tools. A total of 8 full trials were carried out. Of these 8 trials, data from 5 attempts have been the basis of the modeling reported as part of the current article. During the three excluded trials the tool have either experienced plastic deformation (PD) or the chosen cutting data combinations have led to numerical problems in close proximity to the so-called Colding singularity according to Eq. 7. During the trials, more than 50 000 cm\(^3\) chips were produced, equivalent to about 400 kg workpiece material. The total engagement time amounted to 8 hours and 50 minutes with a total length of engagement (spiral distance) \( e_T \) corresponding to 83.5 km. Table 1 below presents selected cutting data for each performed experiment. Other relevant experimental conditions are reported in Table 2. Table 3 shows the values of the model constants in addition to Colding’s constants \( C_i \).
Table 1. Cutting data conditions used during the experimental evaluation. The major cutting edge angle $\kappa = 93^\circ$ and the nose radius $r = 0.8 \text{ mm}$ were used during all experiments.

<table>
<thead>
<tr>
<th>No</th>
<th>Cutting speed $v_c$ [m/min]</th>
<th>Feed rate $f$ [mm/rev]</th>
<th>Depth of cut $a_p$ [mm]</th>
<th>Equivalent chip thickness $h_e$ [mm]</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>220</td>
<td>0.20</td>
<td>2.5</td>
<td>0.164</td>
<td>Excluded</td>
</tr>
<tr>
<td>2</td>
<td>150</td>
<td>0.45</td>
<td>2.5</td>
<td>0.354</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>0.45</td>
<td>2.5</td>
<td>0.354</td>
<td>Excluded</td>
</tr>
<tr>
<td>4</td>
<td>150</td>
<td>0.20</td>
<td>2.5</td>
<td>0.164</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>200</td>
<td>0.30</td>
<td>2.5</td>
<td>0.241</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>300</td>
<td>0.25</td>
<td>2.5</td>
<td>0.203</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>250</td>
<td>0.30</td>
<td>2.5</td>
<td>0.241</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>300</td>
<td>0.20</td>
<td>2.5</td>
<td>0.164</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Set-up data used during the experiments.

<table>
<thead>
<tr>
<th>No</th>
<th>Cutting speed $v_c$ [m/min]</th>
<th>Feed rate $f$ [mm/rev]</th>
<th>Depth of cut $a_p$ [mm]</th>
<th>Equivalent chip thickness $h_e$ [mm]</th>
<th>Comment</th>
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</thead>
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</tr>
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<td>2.5</td>
<td>0.354</td>
<td></td>
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<td>2.5</td>
<td>0.354</td>
<td>Excluded</td>
</tr>
<tr>
<td>4</td>
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<td>0.20</td>
<td>2.5</td>
<td>0.164</td>
<td></td>
</tr>
<tr>
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<td>200</td>
<td>0.30</td>
<td>2.5</td>
<td>0.241</td>
<td></td>
</tr>
<tr>
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<td>300</td>
<td>0.25</td>
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<td>0.203</td>
<td></td>
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<tr>
<td>7</td>
<td>250</td>
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<td>2.5</td>
<td>0.241</td>
<td></td>
</tr>
<tr>
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<td>300</td>
<td>0.20</td>
<td>2.5</td>
<td>0.164</td>
<td></td>
</tr>
</tbody>
</table>

During the conducted experiments the flank wear VB as a function of the engagement time $t$ was recorded at regular intervals in combination with the cutting tool being depicted in 3D through using optical measurement to ensure that no plastic deformation had been obtained. The measurements were made in an optical microscope made by Alicona Infinite Focus. During the trials, the three cutting-force components were continuously recorded, which enabled an assessment of appropriate intervention periods between actual geometric measurements. In Figure 1, the attained flank wear VB for each of the five selected experiments 2, 5, 6, 7 and 8 are plotted as a function of the engagement time $t$. The data has been interpolated to obtain equidistant steps between wear stages ($\Delta VB$).

![Fig. 2](image-url)

Fig. 2. The developments of the tool wear VB as a function of time $t$ for the five selected tests.

The data was linearly interpolated and Fig. 3 shows as an example the measured data and the interpolated data for test number 7. The wear is assumed to start at 0.05 mm which is roughly equal to the tool edge radius $r_{\beta}$ and each data point is linearly interpolated between the two closest measured points.
Fig. 3. The developments of the tool wear VB as a function of time t in test 7 with measured data (red) and interpolated data (blue).

The evaluation of the model is based on the mean linear error $\varepsilon_{err}$ in % according to Eq. 8 between experimentally attained $v_{c, exp}$ and modelled cutting speed $v_{c, mod}$ for each test.

$$
\varepsilon_{err} = \frac{100}{n} \sum_{j=1}^{n} \frac{V_{c,exp,j} - V_{c,mod,j}}{V_{c,exp,j}}
$$

(8)

Determination of Colding constants have been made by using a least squares method through a built-in feature in the software Mathcad 15 based on an algorithm for data fitting developed by Levenberg-Marquardt (Levenburg, 1944; Marquat, 1963). The tool life calculated with the Colding model, Eq. 5, is based on the tool life criterion ranging from $VB_T = 0.10 \rightarrow 0.60$ mm, with calculated constants as found in Table 4. Fig. 3 illustrates the data input for calculation of the 5 Colding constants. Different starting values were evaluated when using the least square method until the lowest error was attained while still keeping $L > 0$.

$$
\begin{align*}
he := & \begin{pmatrix}
0.354 \\
0.241 \\
0.203 \\
0.241 \\
0.164
\end{pmatrix}
\end{align*}
$$

$$
\begin{align*}
v_c := & \begin{pmatrix}
150 \\
200 \\
300 \\
250 \\
300
\end{pmatrix}
\end{align*}
$$

$$
\begin{align*}
T_{30} := & \begin{pmatrix}
10.37 \\
19.10 \\
2.73 \\
4.21 \\
3.45
\end{pmatrix}
\end{align*}
$$

$$
\begin{align*}
VB_{30} := & \begin{pmatrix}
0.3 \\
0.3 \\
0.3 \\
0.3 \\
0.3
\end{pmatrix}
\end{align*}
$$

A
B
C
D

Fig. 4. Example of data input based on 5 tests for determining the Colding constants. $T_{30}$ (column C) is the time used to reach the level of tool wear $VB = 0.30$ mm (column D) for all tests.

Table 3. The five Colding constants and attained model error while using Colding’s equation, Eq. 5, for different flank wear tool wear criterion $VB = 0.10 \rightarrow 0.60$ mm.

<table>
<thead>
<tr>
<th>$VB_T$</th>
<th>K</th>
<th>H</th>
<th>M</th>
<th>$N_0$</th>
<th>L</th>
<th>$\varepsilon_{err}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>5.542</td>
<td>-3.3664</td>
<td>1.5466</td>
<td>0.6055</td>
<td>-0.2013</td>
<td>5.0</td>
</tr>
<tr>
<td>0.15</td>
<td>5.298</td>
<td>-3.0083</td>
<td>2.7518</td>
<td>0.5059</td>
<td>-0.0383</td>
<td>3.5</td>
</tr>
<tr>
<td>0.20</td>
<td>5.417</td>
<td>-1.806</td>
<td>1.5517</td>
<td>1.2741</td>
<td>-0.2139</td>
<td>2.9</td>
</tr>
<tr>
<td>0.25</td>
<td>5.810</td>
<td>-1.6983</td>
<td>0.2404</td>
<td>0.5242</td>
<td>-0.2296</td>
<td>0.4</td>
</tr>
<tr>
<td>0.30</td>
<td>5.815</td>
<td>-1.6918</td>
<td>0.4568</td>
<td>0.4704</td>
<td>-0.2164</td>
<td>1.0</td>
</tr>
<tr>
<td>0.35</td>
<td>5.841</td>
<td>-1.5145</td>
<td>0.9089</td>
<td>0.5020</td>
<td>-0.2364</td>
<td>0.9</td>
</tr>
<tr>
<td>0.40</td>
<td>5.851</td>
<td>-1.3151</td>
<td>1.9241</td>
<td>0.4919</td>
<td>-0.2348</td>
<td>0.9</td>
</tr>
<tr>
<td>0.45</td>
<td>5.888</td>
<td>-0.9259</td>
<td>4.4606</td>
<td>0.4981</td>
<td>-0.2363</td>
<td>0.8</td>
</tr>
<tr>
<td>0.50</td>
<td>5.911</td>
<td>-0.8677</td>
<td>3.1123</td>
<td>0.5043</td>
<td>-0.2381</td>
<td>0.6</td>
</tr>
<tr>
<td>0.55</td>
<td>5.922</td>
<td>-0.9616</td>
<td>2.5709</td>
<td>0.5016</td>
<td>-0.2340</td>
<td>0.6</td>
</tr>
<tr>
<td>0.60</td>
<td>5.941</td>
<td>-0.8291</td>
<td>2.8579</td>
<td>0.5069</td>
<td>-0.2366</td>
<td>0.6</td>
</tr>
</tbody>
</table>
5. RESULTS AND DISCUSSION

Difficulties arise when modelling tool wear with a varying flank wear criterion VB_T. This due to that the coating on the cutting tool may have different degrees of influence depending on the selected tool wear. The main reason for this relationship is the difference in wear resistance between the coating and the tool substrate material. For a given value of the flank wear VB, the wear will break through the coating and thus instead begins to wear on the tool’s substrate. Interaction will occur between the coating and the substrate where the substrate properties will gradually increase in significance as the proportion of the coating reduces in the contact surface, as illustrated in Fig. 5.

![Image of tool wear with coating and substrate](image_url)

Fig. 5. Tool wear in the form of flank wear for VB = 0.31 mm and VB = 0.60 mm.

Fig. 6 shows how the wear propagates for test 7 where the size of the substrate material in contact with the workpiece is plotted as a function of the engagement time t. This clearly shows how the substrate material impact the propagation of tool wear and how the tool wear flattens out after the initial wear is stabilized. The same result was found on all 5 tests.

![Image of substrate wear](image_url)

Fig. 6. The flank wear VB_max and the amount of substrate VB_sub in the tool wear over the engagement time t for test 7.
As the same model is used to describe the wear on both the coating and substrate the model error will decreases as the process stabilizes. This gives an increasing model error with a reduced wear criterion. Further, this study shows that the coating have a higher variation of properties than the substrate material. This variation was believed to have three plausible, dominant causes:

1. The coating is more brittle than the substrate material to some extent resulting in non-wear-related tool deterioration.
2. The coating is more sensitive towards flaking close to the edge line, see red arrows in Fig. 5.
3. In combination with the causes mentioned above, the wear volume is smaller and the local pressure acting on the cutting tool is the highest close to the edge line due to the stagnation point.

An important factor is the model error, and what may be accepted in connection with the use of the model. Typically, models in the industry are expected to have a model error of less than 10%. In practice this means that the cutting speed calculated for a given tool life, or for production of a fixed number of parts must be predicted with an accuracy of at least 10%. When using a cutting speed that is 10% lower than the model suggests this should provide a robust manufacturing process in terms of tool life or attained tool wear.

It can be noticed in this work that for VB ≥ 0.25 mm the model error evens out and is equal to or less than 1% as the coating’s effect on the tool life is marginalized, see Table 3. It can also be noted that depending on the decided wear criterion the constants in the Colding equation will vary whilst still keeping the model error at a minimum.

![Fig. 7. The model error ε_{err} for the Colding equation for different wear criterion VB_{T} = 0.1 \rightarrow 0.6 mm and Colding singularity as a function of different flank wear criterion.](image1)

As described in Eq. 7 the Colding Equation contains a mathematical singularity. The value of the Colding singularity depends on the wear criterion as shown in Fig. 7.

In Fig. 8, 9, and 10 the 5 model constants in the Colding equation are plotted for different tool wear criterion. Also, for all the 5 Colding constants there are similar phenomenon as for the model error at VB ≥ 0.25 mm that can be connected to the coating material of the tool in early stages of the tool wear.

![Fig. 8. The value of the K constant in the Colding equation for different wear criterion.](image2)
For $\text{VB} \geq 0.20$ mm the value of the $K$ constant is an effect of the tool wearing on the coated layer as described in Fig. 5. When the coating is worn through and wear start of the substrate material the $K$ constant stabilizes as the effect of the tool coating decline.

![Fig. 9. The value of constants H and M in the Colding equation for different wear criterion.](image_url)

The behaviour of the $M$ constant for low values of $\text{VB}$ is most likely affected of the Colding singularity effect.

![Fig. 10. The value of constants $N_0$ and $L$ in the Colding equation for different wear criterion.](image_url)

When the model constants are determined on varying wear levels, and wear levels of less than 0.20 mm are involved, the coating that are employed on the tools can affect the result considerably as seen in Fig. 10 for $N_0$ and $L$. This effect combined with the Colding singularity effect is shown clearly in Fig. 9 for $\text{VB} \geq 0.20$ mm were the constants values increase and the settles on stabile values.

6. CONCLUSIONS AND FURTHER WORK

The Colding equation is a well-functioning model for describing the tool detrition for different machining processes. The disadvantage of the model is that it requires at least 5 separate experimental trials where the full wear criterion is reached. This research shows how the 5 constant and the model error changes for different wear criterion to give a deeper understanding of the model.

The general conclusion from this study is the influence of the coating material and its effect on the model constants and model error during the beginning of the tool life. As the tool’s substrate material comes in contact with the workpiece the model error decreases considerably and the expected tool life increases.

Further research is required to further investigate the phenomenon observed as part of the current research. This research can for example be conducted in the following areas:

- Investigation of the possibilities of combining the Colding model with for example the Archard model to gain the strength of both models
- Evaluate effects related to variations in the workpiece material and its effect on the tool life.
- Investigate the influence of cutting fluid during tool life and tool wear modeling.
ACKNOWLEDGMENT

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REFERENCES


